What is light?

- particles?

  Newton, early 1700s:

  geometric optics, reflection, refraction

- Waves?

  Huygens, 1700 + Young, early 1800s

  interference, diffraction, wavelets, superposition

by mid 1850s, wave theory was dominating over particle theory

By end of 1900's everything changed!

<u>Blackbody problem</u>

Imagine a body that is heated up to some temp

=> Keep the temperature constant

=> look at the distribution of light given off

=> there will be different intensities of light at different frequencies (colors)

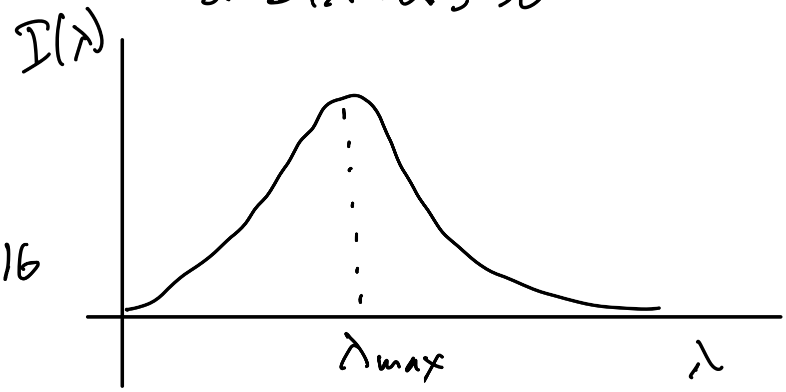
This should be familiar: heat something up, color changes

as temp ↑, freq of light given off ↑ (bluer)

example: Sun surface temp is 5000-6000 K
energy peaks around $\lambda = 550\,nm$

But there is some light intensity all all frequencies

So expect: 1. $I(\lambda)$ peaks at λ_{max}
2. $I(\lambda \to 0) \to 0$
3. $I(\lambda \to \infty) \to 0$



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Ideal body will absorb all frequencies

$\Rightarrow$ so the $I(\lambda)$ given off only depends on temp T

so $I = I(\lambda, T)$

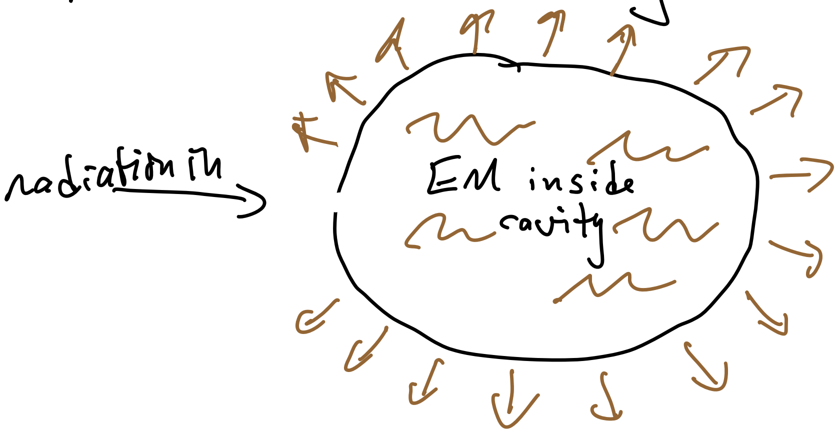
this ideal body is called a "black body"

because it absorbs everything, no reflections

At constant T, it is in equilibrium ($E_{in} = E_{out}$)

Can build such a thing by a hollow cavity with a pin hole for energy to get in (won't escape out)

Then EM radiation out goes thru surface



⇒ Surface radiation is in equilibrium with radiation inside cavity

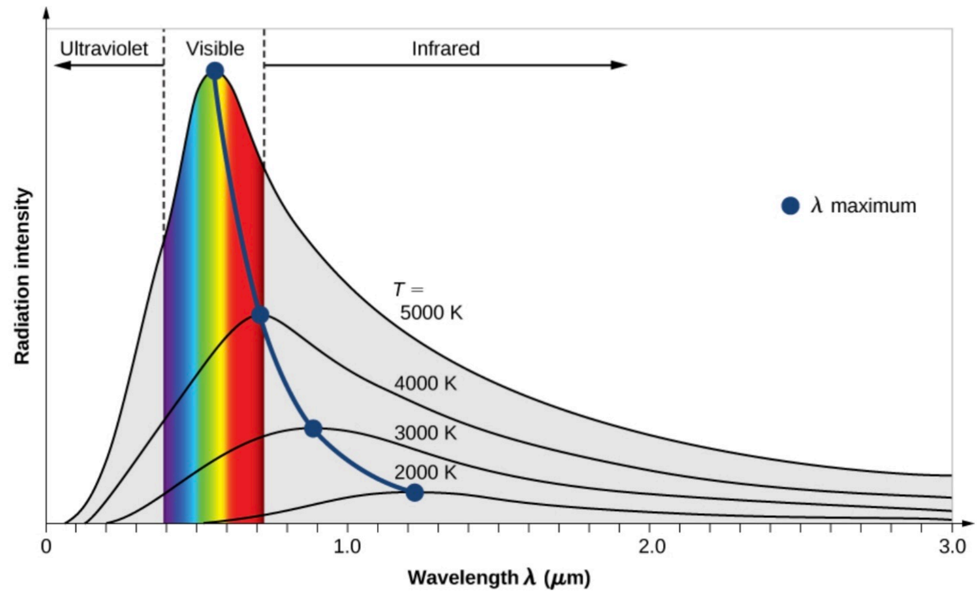
⇒ $I(\lambda, T)$ is intensity vs wavelength for a fixed temperature T

⇒ this is "black body radiation"

By late 1800s, techology allowed accurate measuring of black body spectrum

⇒ especially using telescopes to measure spectrum of stars

Here is what was measured:



Note: intensity peak wavelength depends on temp

as T increases, peak λ decreases

From experiment (plot above) we have Wien's law:

$\lambda_{max} T = 2.898 \times 10^{-3}$ m·K for visible range of λ_{max}

ex: sun's energy peaks at $\lambda \sim 500$ nm yellow

$$T_{sun} = \frac{2.898 \times 10^{-3} \text{ m·K}}{500 \times 10^{-9} \text{ m}} = 5800 \text{ K}$$

Stefan's law: relates power radiated per temp

Power radiated: integrate over all wavelengths

remember Intensity $I = \frac{\text{Power}}{\text{area}}$ so Power $= I \cdot$ area

Stefan's law: $\frac{P_{total}(T)}{A} = \sigma T^4$, $\sigma = 5.67\times10^{-8} \frac{W}{m^2 K^4}$

so $P(T) = \sigma A T^4$ A = surface area of the body radiating

ex: $T_{sun} = 5800\,K$

so $I(5800) = 5.67\times10^{-8} \frac{W}{m^2 K^4} * (5800)^4$

$= 64\times10^6 \frac{W}{m^2}$

Radius $= 7\times10^8\,m$

surface area $A = 4\pi r^2 = 4\pi\times(7\times10^8\,m)^2 = 4.9\times10^{17}\,m^2$

so power radiated $P = I\cdot Area$

$= 64\times10^6 \frac{W}{m^2} * 4.9\times10^{17}\,m^2$

$= 31\times10^{25}\,W$

So... $I(\lambda)$ for different temperatures is VERY weird given what was known about EM radiation:

1. EM energy is in periodic waves

2. for EM waves energy $\propto |\text{amplitude}|^2$

but here,

$\Rightarrow$ as T increases, total energy absorbed is larger

$\Rightarrow$ what is measured is that larger energy (higher T) means shorter wavelengths

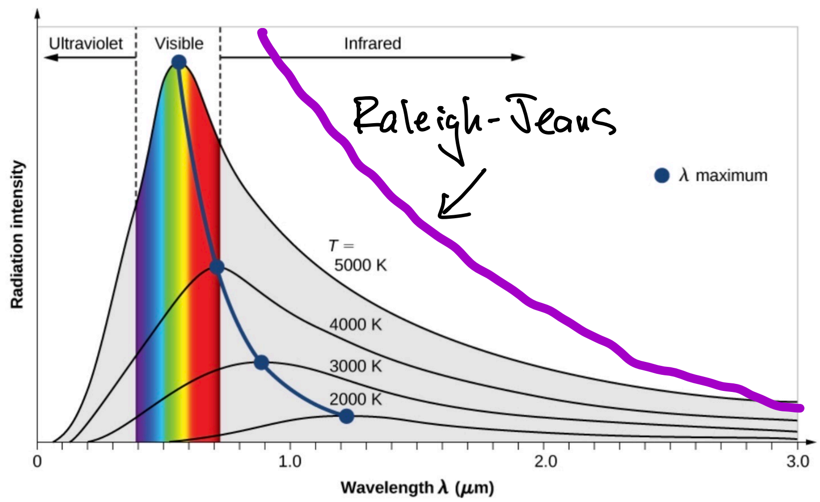
$\Rightarrow$ since $\lambda f = c$, shorter λ = larger f

so larger E = larger f HUH?

What is black body problem in late 19^{th} cent?

how to calculate black body spectrum using what was known about thermodynamics and EM radiation

Raleigh-Jeans: published 1^{st} attempt in 1900



Raleigh-Jeans does not fit!

⇒ especially at shorter wavelengths

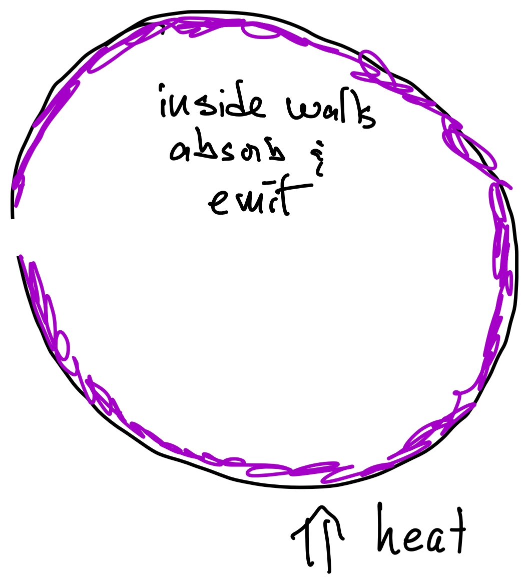
⇒ predicts infinite intensity at short wavelengths = high frequencies

Something is seriously wrong with our basic understanding!

Max Planck : 1900 published a result that <u>DID</u> fit

1. imagine a black body cavity
   ⇒ energy in is from heat
   ⇒ comes to equilibrium, so temp is constant
   ⇒ the <u>only</u> radiation emitted is through the hole

so cavity walls are absorbing E_{in} and emitting E_{out} and $E_{in} = E_{out}$ ⇒ constant temp T



2. the way the walls emit: oscillators in the walls!
   ⇒ atoms, but not known in 1900

Planck said that the oscillators emit EM waves but only at discrete values, <u>not</u> continuous!!

⇒ oscillators emit energy by transitioning from a higher to lower energy

$\Delta E = E_{high} - E_{low}$ are constrained to be proportional to frequency of the radiation

$\Delta E \propto hf$ where h is a constant

$\Rightarrow$ note this is consistent with Wien's law:

heat in $\Rightarrow$ temp and $T \propto \frac{1}{\lambda_{peak}} = \frac{f_{peak}}{c}$

so $E_{radiation} = n\,hf$ n = integer

this is completely new $\rightarrow$ classical E&M say all frequencies are possible

Planck says no, freq of radiation in cavities is discrete (discontinuous)

$\Rightarrow$ Birth of quantum science

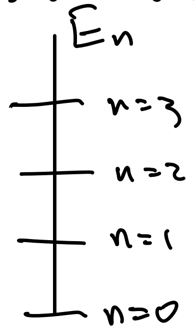
constant $h \equiv$ Planck's constant

measured to be 6.63×10^{-34} J-s

using this, Planck calculate black body spectrum

$$I(\lambda, T) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

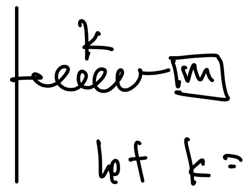
k_B = Boltzmann's constant 1.38×10^{-23} J/K so atoms are oscillators w/ energy "levels"



spacing $\Delta E = E_2 - E_1 = hf$

and spacing is the same for all energy pairs $n+1$ vs n

ex: classical oscillator, mass on spring



$$\omega = \sqrt{\frac{k}{m}}$$

let $k = 100\ N/m$
$m = 1\ kg$

$$\omega = \sqrt{\frac{k}{m}} = 10\ \frac{rad}{sec} = 2\pi f$$

$$\text{so } f = \frac{10}{2\pi}\ Hz = 1.6\ Hz$$

if this were a quantum system, the oscillators would have energy spacing

$$\Delta E = hf = 6.63 \times 10^{-34}\ Js \cdot 1.6\ Hz$$
$$= 10.6 \times 10^{-34}\ J$$

minisule amount of energy

$\Rightarrow$ tells you that quantum effects are hard to see in classical situations

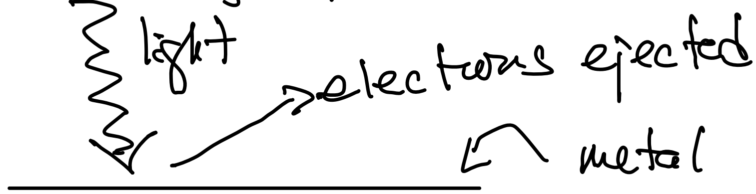
Plank's result was met with skepticism for 5-10 years until:

Photoelectric effect

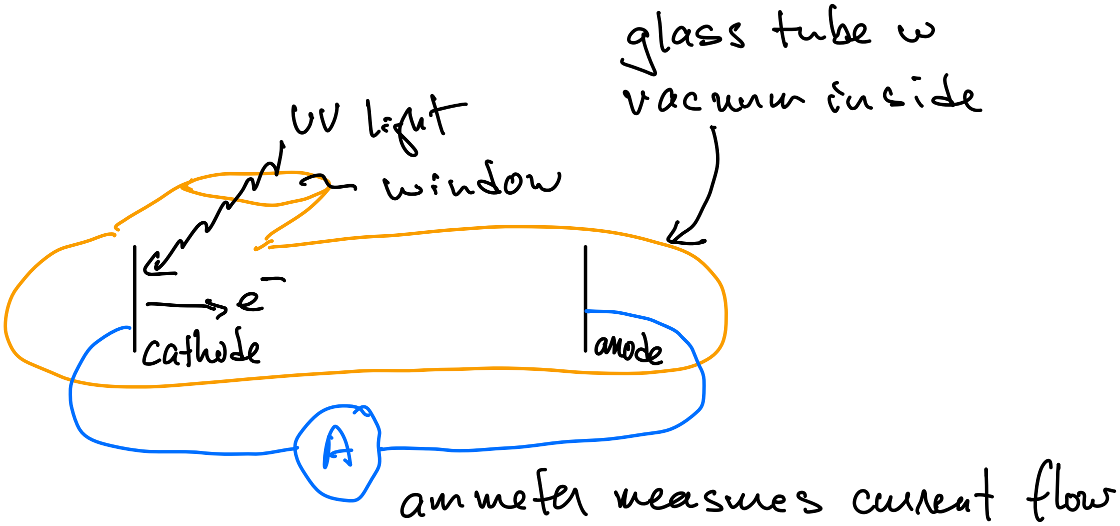
1839, Becquerel discovered photovoltaic effect

→ light on some materials generates voltages

late 1800s, experiments with metals:



Phillip Lenard experiment, 1900 (Nobel 1905)
(1862-1947)



⇒ shine UV light onto cathode, registers a current in the ammeter

saw that: no light, no current

⇒ current in space between electrodes must be same as current in the wires thru meter

can measure intensity of current I_c vs intensity of light I_ℓ

finds that: $I_c \propto I_\ell$

now add battery to circuit

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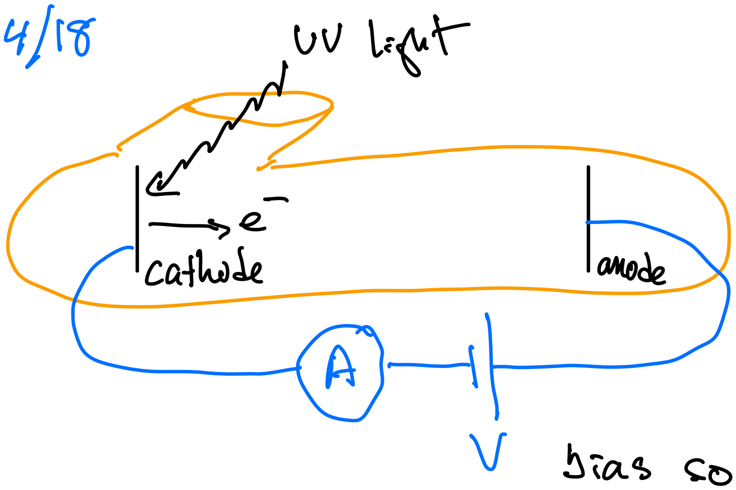


bias so anode is at $+V$ relative to cathode

⇒ this will accelerate electrons to anode

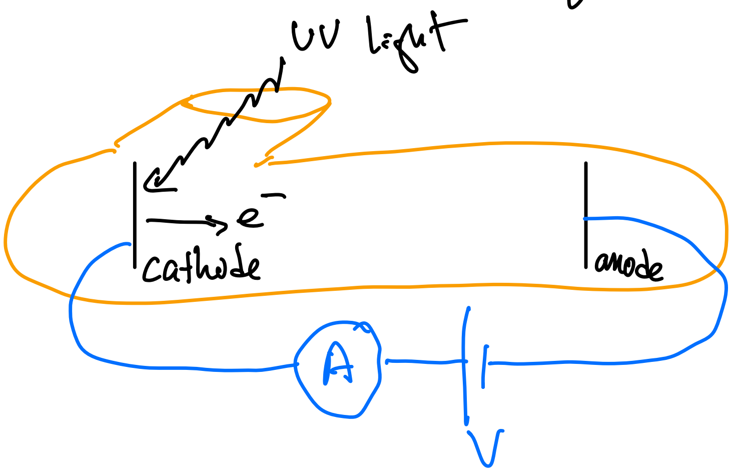
Energy gained by electrons: $\Delta E = eV$

However, Lenard observed that the current thru the ammeter did <u>not</u> change with increasing voltage

but still observed $I_e \propto I_\ell$

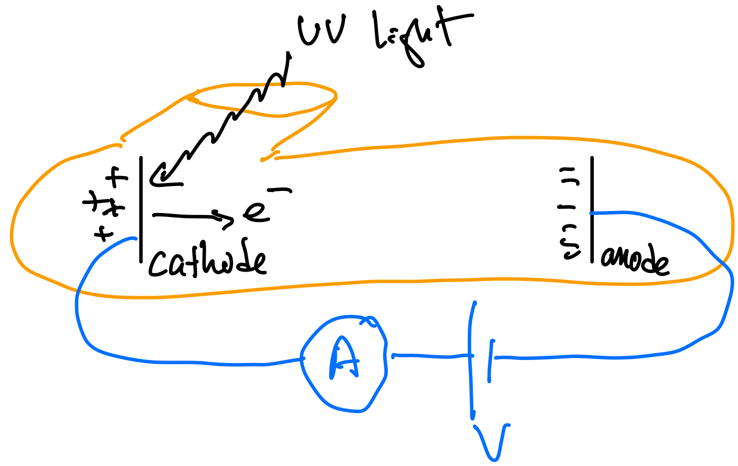
$\Rightarrow$ more light, more current

next reverse polarity of battery



Here, electrons would have to <u>overcome</u> anode repulsion

$\Rightarrow$ Because battery is pumping + charges to cathode!



electrons will need $KE \geq eV$ to get to anode

⇒ Lenard observed that as V increases,
current dropped

but can make up for it by increasing
light current

However: <u>only</u> up to some voltage V_s
(stopping voltage)

and, no matter how much you crank up
light intensity I_ℓ, still no current
as long as $V > V_s$!

⇒ This implies that KE of ejected electrons
is <u>not</u> proportional to light intensity!

Next Lenard increase light frequency and found current I_c turned on!

so $I_c \propto f_{light}$ when $V > V_s$

this implies that KE of electrons ejected is proportional to the light frequency and <u>not</u> light intensity

Problems w/ "classical" picture of photo-electric effect

1. at some stopping voltage, current $\to 0$
   expect current to reappear by increasing light intensity
   result: • current intensity does not depend on light intensity
   • current reappears by raising light frequency

2. it takes some energy ("work function", ϕ) to free an electron from the metal surface

⇒ any electron would have to absorb energy from light intensity to build up enough to get free

⇒ therefore there should be a delay between seeing a current & turning on light

result: no delay seen — current intensity appears immediately!

<u>Note on units</u>

since $h = 6.63 \times 10^{-34}$ J·s is so small, energies in joules will be small

ex: visible light has $\lambda = 500\,\text{nm} = 500 \times 10^{-9}$

$c = \lambda f$ so $f = \frac{c}{\lambda} = \frac{3 \times 10^{8}\,\text{m/s}}{500 \times 10^{-9}\,\text{m}} = 6 \times 10^{14}\,\text{Hz}$

energy associated with this "photon"

$E = hf = 6.63 \times 10^{-34}\,\text{J.s} * 6 \times 10^{14}/\text{s}$

$= 4 \times 10^{-19}\,\text{J}$ small!

Remember from static electricity:

charge Q going thru potential diff of ΔV

picks up or loses energy $E = Q\Delta V$

Q for proton: $e \equiv 1.6 \times 10^{-19}$ Coulombs

if $\Delta V = 1$ volt, $E = 1.6 \times 10^{-14}\,\text{C} \cdot 1\,\text{V} = 1.6 \times 10^{-19}\,\text{J}$

lets define new unit: electron volt =

energy of charge e thru 1 volt

then $1 eV = 1.6 \times 10^{-19} J$ converts Joules to eV

then our 6×10^{14} Hz photon has energy:

$$E = 4 \times 10^{-19} J * \frac{1 eV}{1.6 \times 10^{-19} J} = 2.5 eV$$ nicer unit!

From now on use eV for particles, atoms, etc.

Work function for most atoms that can emit electrons

Na	Sodium	2.36
Al	Alum.	4.06-4.26
Pb	Lead	4.25
Zn	Zinc	3.6-4.9
Fe	Iron	4.6-4.8
Cu	Copper	4.5-5.1
Si	Silicon	4.6-4.9
C	Carbon	5.0
Ag	Silver	4.2-4.7
Ni	Nickel	5.0-5.4
Au	Gold	5.1-5.5

$\Big\}$ units of eV

note all work functions are ~few eV

Resolution: Einstein, 1905

1. Photons are particles
2. Energy of photon $\propto$ photon frequency:

$$E = hf \qquad h \equiv \text{Planck's constant}$$

as proposed by Planck in ~1900

$$h = 6.63 \times 10^{-34}\ \text{J-s}$$

3. each electron absorbs single photon and gains energy $E = hf$

   to get to anode w/ stopping voltage V_s and work function ϕ:

$$KE = hf - \phi \geq eV_s$$

experiment to do

1. shine light w/ some frequency f
2. find V_s that causes zero current
3. vary f, measure V_s, plot:

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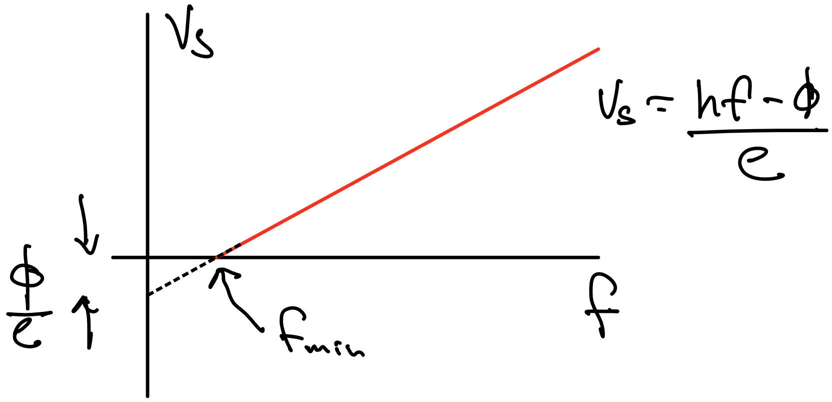


- Positive slope gives $\frac{h}{e}$
- y-intercept gives $V_s(f=0) = -\phi/e$

Verified! Theory fit the data!

ex: light w/ $\lambda = 400nm$

$c = \lambda f$ so $f = c/\lambda$

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,J\text{-}s * 3\times10^{8}\,m/s}{400\times10^{-9}\,m}$$

$$= 5.0\times10^{-19}\,J \quad \text{small!}$$

then in eV, $E = 5\times10^{-19}\,J * \frac{1eV}{1.6\times10^{-19}\,J} = 3.1\,eV$

so 400nm is not going to eject electrons except in sodium

=> usually we want to go from wavelength to energy

$E = hf$ and $f = c/\lambda$

so $E = \frac{hc}{\lambda}$

we want to use λ in nm and E in eV
so calculate hc in eV-nm

$$hc = 6.63\times10^{-34}\,J\cdot s * 3\times10^{8}\,\frac{m}{s} = 1.989\times10^{-25}\,J\text{-}m$$

$$hc = 1.989\times10^{-25}\,J\text{-}m * \frac{1eV}{1.6\times10^{-19}J} * \frac{1nm}{10^{-9}m} = 1243\,eV\text{-}nm$$

use $\boxed{hc = 1243\,eV\text{-}nm}$

ex: light w/ wavelength 600 nm has energy

$$E = \frac{hc}{\lambda} = \frac{1243\,eV\text{-}nm}{600\,nm} = 2.1\,eV$$

ex: what is maximum wavelength of light that can eject electrons from Na and Au?

for Na, $\phi = 2.36\,eV$
Au $\phi = 5.2\,eV$

incident photon energy

KE of ejected electrons: $KE = E_\gamma - \phi$ energy needed to get out of material

minimum $KE = 0$ so $E_\gamma - \phi = 0$

$$E_\gamma = hf_{min} = \frac{hc}{\lambda_{max}} = \phi \qquad \lambda_{max} = \frac{hc}{\phi}$$

for Na $\lambda_{max} = \frac{1243\,eV\text{-}nm}{2.36\,eV} = 540\,nm$ green

for Au $\lambda_{max} = \frac{1243\,eV\text{-}nm}{5.2\,eV} = 240\,nm$ UV

ex: $\lambda = 300$ nm photons hit Na

what is KE of electrons ejected?

$$KE = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1243\,eV\text{-}nm}{300\,nm} - 2.36\,eV$$

$$= 4.14\,eV - 2.36\,eV = 1.78\,eV$$

KE = 1.78 eV

if mass of electron is 9.1×10^{-31} kg, what is velocity?

rest energy of electrons $E_0 = m_0c^2 = 511$ keV

here $KE \ll E_0$ (kinetic energy $\ll$ rest mass energy)

so use non-relativistic formula

(easier than relativity but will get the same answer)

$$KE = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\,KE \cdot c^2}{mc^2}} = c\sqrt{\frac{2KE}{mc^2}}$$

$$= \sqrt{\frac{2 \times 1.78\,eV}{511 \times 10^3\,eV}} \cdot c = 2.6 \times 10^{-3}c$$

$$= 7.9 \times 10^5\,m/s$$

Photon energy & momentum

$E = hf$ and $c = f\lambda$ so $f = c/\lambda$

so $E = \dfrac{hc}{\lambda}$

Special relativity: $E^2 = (pc)^2 + (m_0 c^2)^2$

from other experiments: $m_0 = 0$ for photons (γs)

so $E = pc$

$p = E/c = \dfrac{hc}{\lambda}/c = \dfrac{h}{\lambda}$

$$\boxed{\left.\begin{aligned} E &= \frac{hc}{\lambda} \\ p &= \frac{h}{\lambda} \end{aligned}\right\} \quad E = pc}$$ for photons

note: $hc = 6.63\times10^{-34}\,\mathrm{J\text{-}s} * 3\times10^{8}\,\mathrm{m/s}$

$= 1.989\times10^{-25}\,\mathrm{J\text{-}m} * \dfrac{1\,\mathrm{eV}}{1.6\times10^{-19}\,\mathrm{J}}$

$= 1.243\times10^{-6}\,\mathrm{eV\text{-}m}$

$= 1.243\,\mathrm{eV\text{-}\mu m}$

$= 1243\,\mathrm{eV\text{-}nm}$

This lets you convert from wavelength to energy

ex: $\lambda = 400nm$

$$E = hf = \frac{hc}{\lambda} = \frac{1243\ eV\text{-}nm}{400\ nm} = 3.1eV$$

ex: laser pointer w/output 5mW emits red light, $\lambda = 650\ nm$

energy of each photon $E = \frac{hc}{\lambda} = \frac{1243 eV\text{-}nm}{650 nm}$

$= 1.9eV$

<u>Compton scattering & compton effect</u>

or Light scattering (off of other particles like electrons in air molecules)

Wave prediction:

$\Rightarrow$ light w/freq f hits a charged particle (eg molecule in air)

$\Rightarrow$ charged particle will feel $\vec{E}$ from light and will oscillate like an antenna

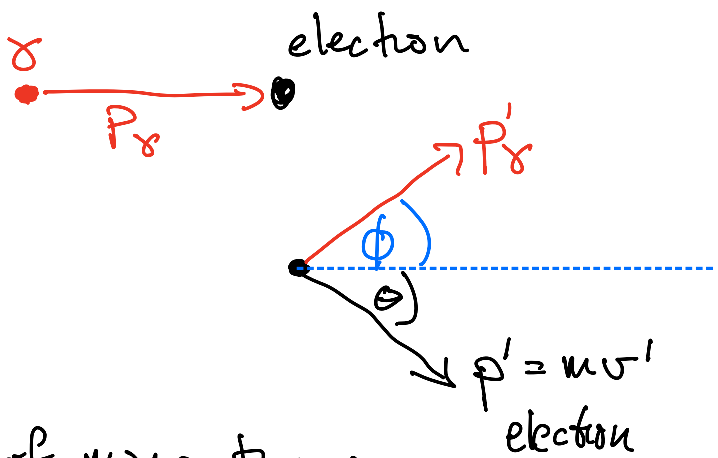
⇒ that will send out EM waves from oscillating charged particle at same f

so wave picture says absorption & radiation are at same freq f

Photon model:

⇒ photons are particles with energy $E_\gamma = hf$

Before:



After:

conservation of momentum:

$$p_\gamma = p'_\gamma \cos\phi + p'\cos\theta$$ x component

$$p_\gamma - p'_\gamma\cos\phi = p'\cos\theta$$

$$p'_\gamma \sin\phi = p'\sin\theta$$ y component

so $$(p_\gamma - p'_\gamma\cos\phi)^2 + p'^2_\gamma \sin^2\phi = p'^2$$

$$p_\gamma^2 + p'^2_\gamma\cos^2\phi - 2p_\gamma p'_\gamma\cos\phi + p_\gamma^2\sin^2\phi = p'^2$$

so $p'^2 = p_\gamma^2 + p_\gamma'^2 - 2p_\gamma p_\gamma' \cos\phi$ (1)

also conservation of energy:

Before: $E_{tot} = E_\gamma + m_e c^2$

After: $E_{tot} = E_\gamma' + \left((m_e c^2)^2 + (p'c)^2\right)^{1/2}$

so $E_\gamma + mc^2 = E_\gamma' + \left((mc^2)^2 + (p'c)^2\right)^{1/2}$

$(E_\gamma + mc^2 - E_\gamma')^2 = (mc^2)^2 + (p'c)^2$

$((E_\gamma - E_\gamma') + mc^2)^2 = (E_\gamma - E_\gamma')^2 + 2mc^2(E_\gamma - E_\gamma') + (mc^2)^2$

$= (mc^2)^2 + (p'c)^2$ (2)

so $(E_\gamma - E_\gamma')^2 + 2mc^2(E_\gamma - E_\gamma') = (p'c)^2$

for photons: $E^2 = (m_0c^2)^2 + (pc)^2$ but $m_0 = 0$ for γ's

so $E = pc$ (photons only)

divide (2) by c^2:

$$p'^2 = \frac{(E_\gamma - E_\gamma')^2}{c^2} + 2m(E_\gamma - E_\gamma')$$

$$= (p_\gamma - p_\gamma')^2 + 2mc(p_\gamma - p_\gamma')$$

$$= p_\gamma^2 - 2p_\gamma p_\gamma' + p_\gamma'^2 + 2mc(p_\gamma - p_\gamma')$$

from (1)

$$p'^2 = p_\gamma^2 + p_\gamma'^2 - 2p_\gamma p_\gamma' \cos\phi$$

subtract:

$$0 = -2p_\gamma p_\gamma' + 2mc(p_\gamma - p_\gamma') + 2p_\gamma p_\gamma' \cos\phi$$

$$2p_\gamma p_\gamma'(1-\cos\phi) = 2mc(p_\gamma - p_\gamma')$$

for photons: $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$ $\quad (c = f\lambda)$

so $\frac{2h^2(1-\cos\phi)}{\lambda\lambda'} = 2mch\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

mult by $\frac{\lambda\lambda'}{2}$: $h^2(1-\cos\phi) = mch(\lambda' - \lambda)$

so $\boxed{\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)}$

this predicts the shift in wavelength for scattering photons ("Compton scattering")

$\Rightarrow$ Verified by experiment!

note: $\frac{h}{mc} = \frac{hc}{mc^2} \Rightarrow$ units of length λ_c

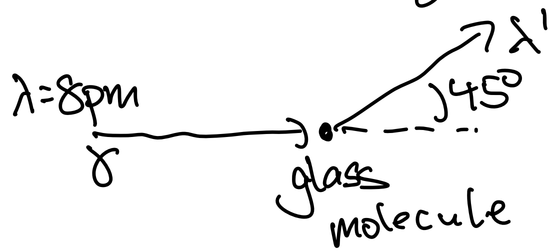
this is called the Compton wavelength

$\Rightarrow$ represents limit of precision for knowing a particle's position (more below)

for electron $\lambda_c = \frac{hc}{m_e c^2} = \frac{1243\,\text{eV-nm}}{511 \times 10^3\,\text{eV}} = 2.43 \times 10^{-3}$ nm we usually use pm, pico-meters, 1 pm = 10^{-12} m

so λ_c for electron is 2.43 pm

ex: $\lambda = 80$ pm x-ray scatters off of electrons in glass
what is λ' of scattered x-ray at $45°$?



Compton scattering of x-rays from electrons

$$\Delta\lambda = 2.43 * (1 - \cos\theta)$$
$$= 2.43 * (1 - \cos 45°) = 0.71 \text{ pm}$$

$\Delta\lambda = \lambda' - \lambda = 0.71$ pm so $\lambda' = \lambda + 0.71$
$= 80.71$ pm

<u>note on how to determine whether relativistic or not</u>

$$E = \gamma m_0 c^2$$
$$KE = (\gamma - 1) m_0 c^2$$

relativistic: $\beta \to 1$, γ large, $E >> m_0 c^2$

non-relativistic: $\beta \to 0$, $\gamma \to 1$, so $KE << m_0 c^2$

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Light intensity I in photon picture

For light, power = $\frac{\text{energy}}{\text{sec}}$

intensity = $\frac{\text{power}}{\text{area}}$

intensity I = flux, units of $\frac{\text{Power}}{\text{area}} = \frac{\text{Energy/sec}}{\text{area}}$

is the rate of energy flow per time thru an area
What does this mean in photon picture?

let R = #γ's/sec photons per second

if all the photons have $E = hf$ then
energy/sec (power) is

$$\frac{\text{energy}}{\text{sec}} = \frac{\text{photons}}{\text{sec}} * \frac{\text{energy}}{\text{photon}}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$P = R * hf$$

or $R = \frac{P}{hf}$ # of photons/sec

ex: laser pointer has 5mW power and $\lambda = 650\,\text{nm}$

$$R = \frac{P}{hf} = \frac{P}{hc/\lambda}$$

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$$= \frac{5\times10^{-3}\,J/s}{1243\,eV/nm} * 650\,nm * \frac{1\,eV}{1.6\times10^{-19}\,J}$$

$$= 1.6\times10^{16}\ \gamma/sec$$

ex: sun intensity is ~ 1400 W/m² at top of atmosphere

if all those γ's were ~400 nm, how many photons hit an area of 1 m² per sec?

$$1400\,\frac{W}{m^2} * 1\,m^2 = 1400\,W = 1400\,\frac{J}{s}$$

total energy in $J = \#\gamma\text{'s} * \frac{\text{energy}}{\text{photon}} = N_\gamma \cdot hf$

$$= N_\gamma \frac{hc}{\lambda} = N_\gamma \cdot \frac{1243\,eV\text{-}nm}{400\,nm}$$

$$J = N_\gamma \cdot 3.1\,eV$$

$$1400\,\frac{J}{s} = \frac{N_\gamma}{s} \cdot 3.1\,eV \quad \text{so} \quad \frac{N_\gamma}{s} = \frac{1400\,J}{3.1\,eV} * \frac{1\,eV}{1.6\times10^{-19}\,J}$$

$$= 2.8\times10^{21}\ \gamma/s$$

lots of γ's per sec!

Atomic spectra

Heat material → gives off light

⇒ Can use diffraction grating to carefully measure wavelengths of emitted light

If substance is a gas of one particular element, light emitted has a very restricted set of wavelengths : atomic spectra

e.g.

Big puzzle!

What was known about atoms ~ 1900:

1897 : Thompson measured $\frac{q}{m}$ of atoms

1909 : Millikan measures q ⇒ this allowed determination of atomic masses

⇒ mass atoms ≫ mass all electrons

The nucleus

Up until 1910, theory of nucleus consisted of

1. electrons, negatively charged
   ⇒ cathode rays were readily observed
2. some positive charge
   ⇒ atoms observed to be neutral

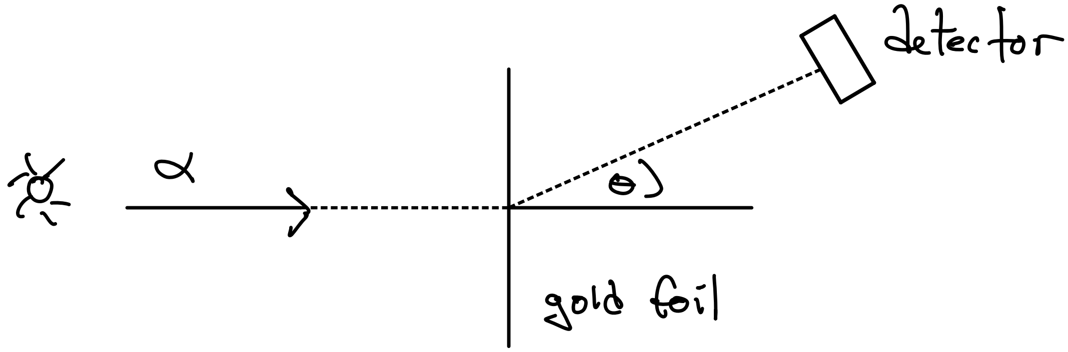
"Plum pudding" model:

electrons (plums) embedded somehow in positively charged "pudding"

Ernest Rutherford, 1911, Univ of Manchester in England

had students do experiment:
particles called "alphas" bombard target made of gold or silver

α's: positive charge
come from naturally radioactive elements



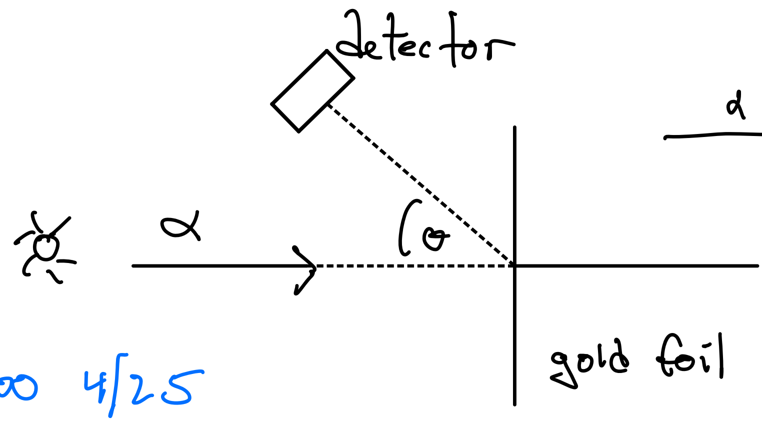
- α's will penetrate gold foil if its thin enough
- place detector at angle Θ, record α hits
- vary Θ

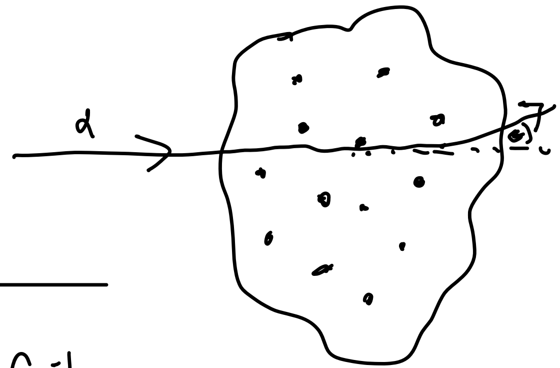
expectation:

- α's would scatter weakly thru the "plum pudding" of pos charge
- electrons have very small mass compared to α's $\left(\frac{m_e}{m_\alpha} \sim \frac{1}{7000} \right)$ so all scattering is due to the positively charged part of the atom
- since atom is neutral, scattering will be when α is inside atom

$\Rightarrow$ such scattering experiments allow probing internal structure of things

Result was surprising: some back scattering!





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This implied:

- "plum pudding" was not correct
- atom must have "hard center"
  $\Rightarrow$ solar system model, hard positively charged nucleus w/ electron in "orbit"

ex: $m_\alpha = 6.64 \times 10^{-27}$ kg

$$m_\alpha c^2 = 6.64 \times 10^{-27} \text{ kg} * \left(3\times 10^8 \frac{m}{s}\right)^2 * \frac{1 eV}{1.6\times 10^{-19} J}$$

$$= 3.735 \times 10^9 eV = 3.735 \text{ GeV}$$

KE of α from radioactive Ra(226) is 4.7 MeV

since $KE \ll mc^2$ α are non relativistic
so do not need relativity

ex: $KE = 4.7 MeV = \frac{1}{2} mv^2 = \frac{1}{2} mc^2 \beta^2$

$4.7\times10^6 eV = \frac{1}{2}(3.735\times10^9 eV)\cdot\beta^2$

$$\beta = \sqrt{\frac{2 * 4.7\times10^6}{3.735\times10^9}} = 0.0502 \ll 1$$

$$v = \beta c = 0.0502 * 3\times10^8 \frac{m}{s} = 15.1\times10^6 \frac{m}{s}$$

Rutherford's theory: electrons are in "orbit" around nucleus

Problems
- that's like an antenna $\rightarrow$ electrons in orbit should radiate energy and fall into nucleus
  $\Rightarrow$ but atoms are stable!
- as electrons spiral in, charge of atom should change
  $\Rightarrow$ it doesn't
- any emitted light should have continuous wavelengths
  $\Rightarrow$ atomic spectra are <u>not</u> continuous

Rutherford's "solar system" model is consistent with scattering experiment results

$\Rightarrow$ but not the rest of EM theory!

1913 Niels Bohr

- atomic spectra tell us that excited atoms emit light w/ specific wavelengths

so $E = \frac{hc}{\lambda} = hf$ must have specific (not continuous) energies

so atomic excitation must mean:

1. excited atom has energy E_n (n is integer)
2. de-excites, and emits γ of some frequency
3. new energy is E_m

$$E_n - E_m = hf \quad \text{energy conservation}$$

$\Rightarrow$ heating atom drives it into high energy state

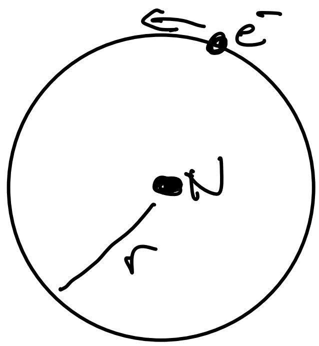
$\Rightarrow$ de-excite transitions back & emits γ

so energy levels of atoms must be <u>quantized</u>

$\Rightarrow$ this is revolutionary!!

Bohr model using quantization ideas

apply wave nature of electrons

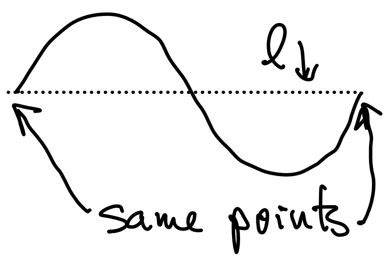


electron in orbit around nucleus

if electrons are in orbit w/radius r then circumference $l = 2\pi r$

if electrons are waves then circumference has to be an integer number of wavelengths:

$$l = n\lambda_n = 2\pi r$$



this means radius is quantized:

$$2\pi r_n = n\lambda_n \Rightarrow \lambda_n = 2\pi r_n / n$$

and deBroglie holds: $p = \frac{h}{\lambda}$

so if $\lambda = \lambda_n$ then p is also quantized

$$p_n = mv_n = \frac{h}{\lambda_n} = \frac{h}{2\pi r_n/n} = \frac{nh}{2\pi r_n}$$

$$mv_n r_n = \frac{hn}{2\pi} = \hbar n \quad (\hbar \equiv h/2\pi)$$

$mv_n r_n \Rightarrow$ angular momentum L_n

$\boxed{L_n = n\hbar}$ angular momentum is quantized

angular acceleration: $\frac{mv^2}{r}$

Force due to EM attraction:

$$F_{EM} = \frac{1}{4\pi\varepsilon_0}\frac{q_e q_N}{r_n^2} = \frac{mv^2}{r_n} \Big\}$$ centripetal force on electron

let $q_e = -1e$ electron

$q_N = +1e$ hydrogen

magnitude of force $F_{EM} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}$ inward (attracts)

this holds electron in orbit

centripetal acceleration $a_c = \frac{v^2}{r_n}$

centripetal force $ma = \frac{mv^2}{r_n} = F_{EM}$ from EM attraction

so $\frac{e^2}{4\pi\varepsilon_0 r_n^2} = \frac{mv^2}{r_n}$

or $mv^2 r_n = \frac{e^2}{4\pi\varepsilon_0}$ multiply both sides by mr_n

$$m^2 v^2 r_n^2 = \frac{m r_n e^2}{4\pi\varepsilon_0}$$

$mvr = p \cdot r =$ angular momentum $L = mvr$

and $L = L_n = n\hbar$ from above

so $m^2 v^2 r_n^2 = L^2 = n^2 \hbar^2 = \frac{m r_n e^2}{4\pi\varepsilon_0}$

solve for $r_n = n^2 \cdot \frac{4\pi\varepsilon_0 \hbar^2}{me^2}$

multiply right side by c^2/c^2

$$r_n = n^2 \cdot \frac{4\pi\varepsilon_0 \hbar^2 c^2}{e^2 \cdot mc^2}$$

$\hbar c = \frac{hc}{2\pi} = \frac{1243 \text{ eV-nm}}{2\pi} = 197.3 \text{ eV-nm}$

$mc^2 = 511 \text{ keV}$ electron rest energy

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$\hbar c$ has units of Energy * Length
mc^2 " " " Energy
r_n " " " Length

so $\frac{e^2}{4\pi\epsilon_0 \hbar c}$ has no units, is a dimensionless number

$\frac{1}{4\pi\epsilon_0} = 9\times10^9\ N/C$

so $$\frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{(1.6\times10^{-19}\,C)^2}{\frac{6.6\times10^{-34}}{2\pi}\,J\text{-}s * 3\times10^8\,\frac{m}{s}} * 9\times10^9\,Nc^2/m^2$$

$$= \frac{1}{137}$$

call this $\alpha = \frac{1}{137}$ "fine structure constant"

so $r_n = n^2 \cdot \frac{\hbar c}{\alpha m_e c^2}$ $\qquad \hbar c = \frac{hc}{2\pi} = \frac{1243\ eV\text{-}nm}{2\pi}$

$= 197.3\ eV\text{-}nm$

$$\frac{\hbar c}{\alpha m_e c^2} = \frac{1973\ eV\text{-}nm \cdot 137}{511\times10^3\,eV}$$

$= 0.053\ nm = 0.53\,\mathring{A}$ (angstrom $1\mathring{A} = \frac{1}{10}nm$)

define "Bohr radius" $a_0 = 0.53\,\mathring{A}$

then $r_n = n^2 a_0$ for hydrogen

ground state $n=1$ $\quad r_1 = a_0$ radius of $\bar{e}$ in ground state

$n=2$ $\quad r_2 = 4a_0$

$n=3$ $\quad r_3 = 9a_0$

etc up to $n=\infty$ where electron is "free" (no longer bound to proton)

so angular momentum and radii of electron in H atom are quantized

$\Rightarrow$ what about energy?

is electron relativistic?

$L_n = n\hbar = m v r_n$ angular momentum

in ground state $r_1 = 1^2 \cdot a_0 = 0.53\,\text{Å}$

write $\hbar = m v r_1$

and so $\hbar c = mc \cdot v r_1$

$$= mc^2 \cdot \frac{v}{c} \cdot r_1$$

so $\dfrac{v}{c} = \dfrac{\hbar c}{mc^2 r_1} = \dfrac{197.3\,\text{eV-nm}}{511 \times 10^3\,\text{eV} \cdot 0.053\,\text{nm}}$

$= 7.3 \times 10^{-3} \ll 1$ so

<u>non</u> relativistic

energy $E = \underbrace{KE}_{\frac{1}{2}mv^2} + \underbrace{PE}_{\text{EM potential energy}}$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2}{r}\cdot r$$

but we see above $\frac{mv^2}{r} = F_{EM} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}$

so $KE = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r_n^2}\cdot r_n = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r_n}$

$PE = \frac{-e^2}{4\pi\varepsilon_0 r_n}$ (last semester)

so $E = KE + PE = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r_n} - \frac{e^2}{4\pi\varepsilon_0 r_n}$

$$= \frac{-e^2}{2\cdot 4\pi\varepsilon_0 r_n}$$

substitute $r_n = n^2\cdot\frac{\hbar c}{\alpha mc^2}$

$$E = -\frac{e^2}{2\cdot 4\pi\varepsilon_0}\frac{\alpha mc^2}{n^2\hbar c} = -\frac{1}{2}\underbrace{\frac{e^2}{4\pi\varepsilon_0\hbar c}}_{\alpha}\frac{\alpha\cdot mc^2}{n^2}$$

$$E_n = \frac{1}{2}\frac{\alpha^2 mc^2}{n^2}$$

$$\frac{\alpha^2 mc^2}{2} = \frac{1}{2}\cdot\left(\frac{1}{137}\right)^2 \cdot 511\times10^3 eV = 13.6 eV$$

note $13.6eV \ll m_e c^2$ so non relativity ok

so $E_n = \frac{-13.6eV}{n^2}$

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$E_n < 0$ because it takes energy to free the electron

need 13.6eV added to hydrogen to free electron (and the liberated electron will have no KE!)

add more than 13.6 and e^- will have KE

13.6 is the work function of hydrogen!

lowest energy is called the "ground state"

ex: atomic spectra of some element has 3 longest wavelengths: 1240 nm, 620nm, 414 nm

this means there are 3 transitions

$\Delta E_0, \Delta E_1, \Delta E_2$

let $E_0 = 0.0\,eV$

$\Delta E = hf = \frac{hc}{\lambda}$ so 1240 nm has smallest energy transition

$\Delta E_0 = \frac{hc}{1240\,nm} = \frac{1243\,eV\text{-}nm}{1240\,nm} \sim 1\,eV$

$\Delta E_1 = \frac{hc}{620\,nm} = \frac{1243\,eV\text{-}nm}{620\,nm} \sim 2\,eV$

$\Delta E_2 = \frac{hc}{414\,nm} = \frac{1243\,eV\text{-}nm}{414\,nm} \sim 3\,eV$

what fits: E_0 = ground state = 0.0 eV

transition from $E_1 \to E_0$

$E_2 \to E_0$

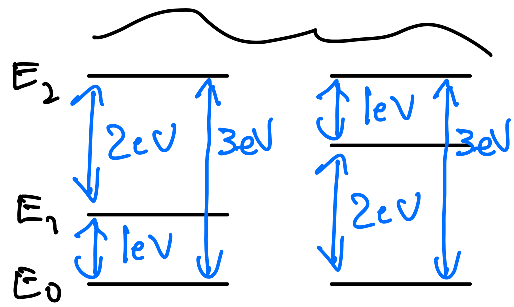
$E_2 \to E_1$

$E_2 \to E_0 = E_2 \to E_1 + E_1 \to E_0$

so $3\,eV = 2\,eV + 1\,eV$

or $3\,eV = 1\,eV + 2\,eV$

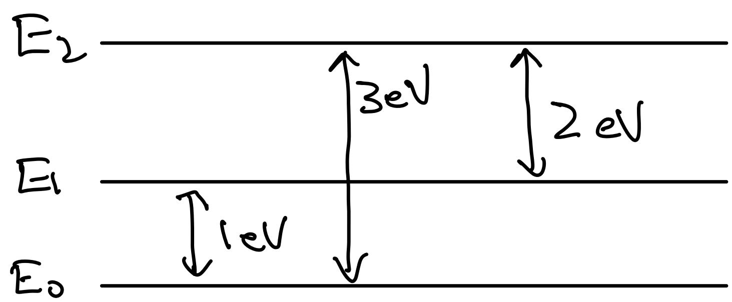
both are ok



break ambiguity by measuring what γ's can be absorbed by ground state

ex: say you measure that ground state can change by absorbing 1240 nm (1eV) or 414 nm (3eV) γ but not 620 nm (2eV)

⇒ this tells you the energy levels look like this



⇒ the 620nm γ has no effect on atom in ground state

But if you excite atom with 414nm γ, it can go into E_2 state and decay to E_1 emitting 620nm γ

Bohr theory recap

1. Atoms have "states"
2. each state has a different energy, radius of electron, angular momentum
3. states are "quantized", not continuous
   "n" for each state, $n=0$ is ground state

quantized properties

$L_n = n\hbar$ angular momentum, $\hbar = \frac{h}{2\pi}$

$r_n = n^2 a_0$ radius a_0 is Bohr radius

$a_0 = 0.529\,\mathring{A} = 0.0529\text{ nm}$

$a_0 = \hbar c/\alpha m_e c^2$ m_e = electron mas

$$E_n = -\frac{13.6\text{eV}}{n^2}$$

Atomic spectra:

ground state H is $E_1 = -13.6\text{eV}$

1st excited state $E_2 = -\frac{13.6\text{eV}}{2^2} = -3.4\text{ eV}$

to excite H from E_1 to E_2 add energy

$$E_{in} = E_2 - E_1 = -3.4 - (-13.6) = 10.2\text{eV}$$

to excite using photon, $E_{in} = E_\gamma = \frac{hc}{\lambda}$

so need photon $\lambda = \frac{hc}{E_\gamma} = \frac{1243\text{eV-nm}}{10.2\text{eV}}$

$= 121.9\ nm$ (UV γ)

$\Rightarrow$ Any photon of wavelength λ that satisfies:

$$\frac{hc}{\lambda} = E_m - E_n \qquad m, n \text{ integers}$$

will be absorbed by atom

Emission: can happen when atom goes from n to m state $(n > m)$

transitions: $\frac{hc}{\lambda} = E_{n_i} - E_{n_f} \qquad n_f < n_i$ (eg. $2 \to 1$)

$$= -\frac{13.6eV}{n_i^2} + \frac{13.6eV}{n_f^2}$$

$$= 13.6eV\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

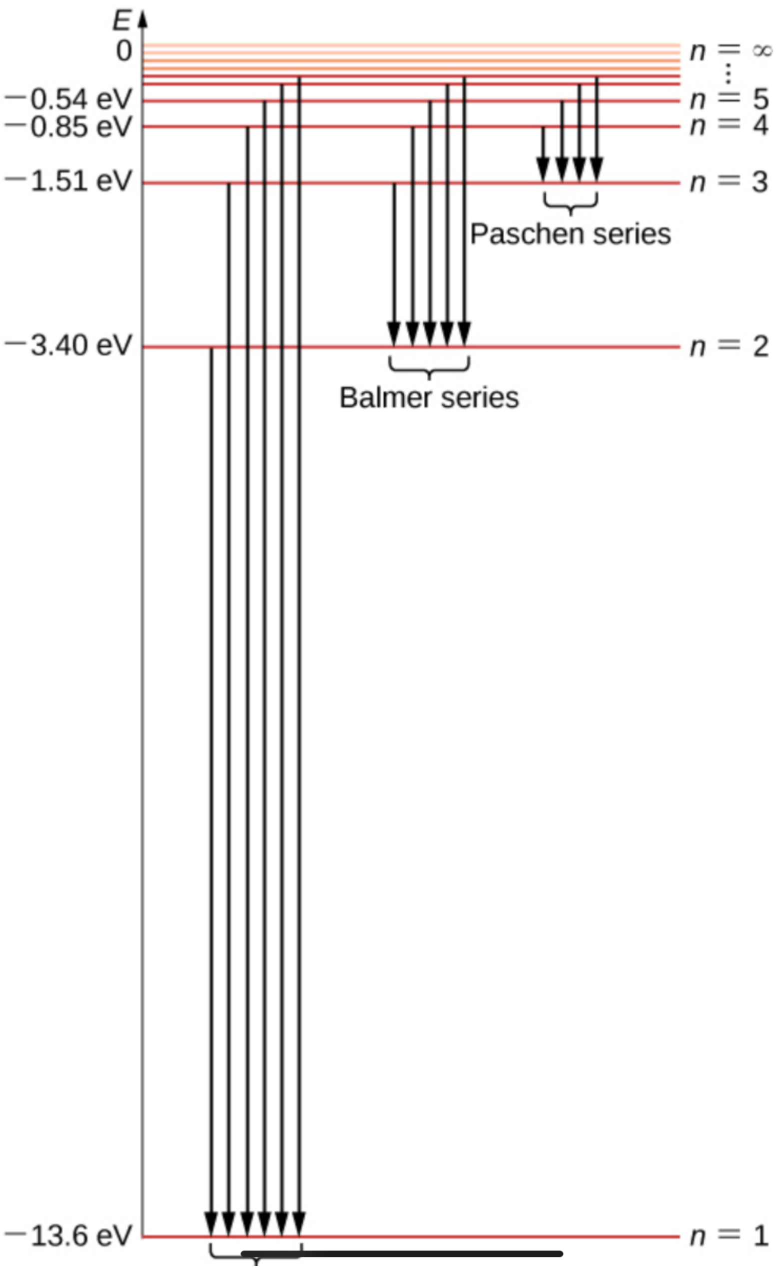
write $\frac{1}{\lambda} = \frac{13.6}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\frac{13.6}{hc} = \frac{13.6eV}{1243eV\text{-}nm} = 1.09 \times 10^{-2}\ nm$$

define $R = 1.09 \times 10^{-2}\ nm$ "Rydberg constant"

then $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ is wavelength of γ emitted

when H transitions from n_i to n_f


E
0
−0.54 eV
−0.85 eV
−1.51 eV
−3.40 eV
−13.6 eV
n = ∞
n = 5
n = 4
n = 3
n = 2
n = 1
Paschen series
Balmer series

Waves & Particles?

Light diffracts & interferes ⇒ waves

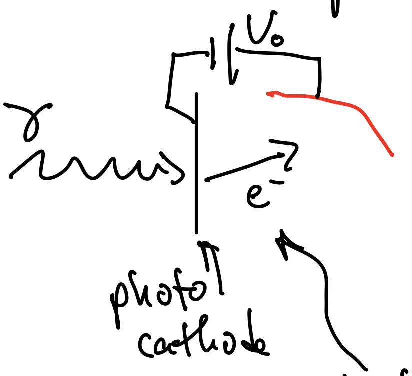
Light consists of photons ⇒ particles?

Which is correct? <u>Both</u>

If you do an expiment to measure wave-like properties, it will behave like a wave.

Same for particle-like properties

Photo-multiplier tube: detects photons:

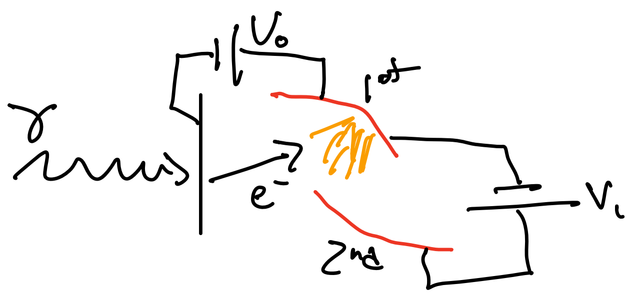


electron is accelerated to 1st stage, which is at a $+V_0$ potential

$\bar{e}$ gains energy eV_0 and hits plate

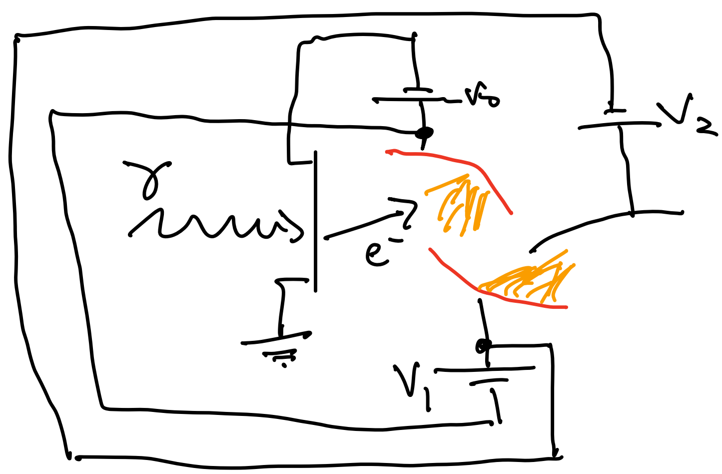
this kicks off more electrons that share eV_0 energy

⇒ next place 2nd stage w/ potential V_1 above 1st accelerate electrons from 1st stage to 2nd stage



secondary electrons each gain energy eV_1 and hit 2nd stage

2nd stage produces tertiary electrons that head to 3rd stage that is at V_2 above 2nd stage



ect. Each stage produses N x previous stage.

if you have n stages, final current of electrons:

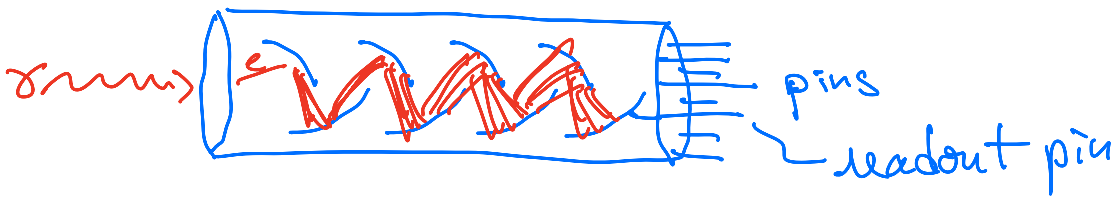
$$I = \frac{\Delta Q}{\Delta t} = \frac{N^n \cdot e}{\Delta t}$$

these are fast, $\Delta t \approx 10\,ns$

if you start w/ 1 photon and have $N=10$ produced at each stage in $10\,ns$:

$$I = 10^{10} \cdot \frac{1.6\times10^{-19}\,C}{10\times10^{-9}\,s} = 0.16\ \text{Amps}$$

so it's a photo-multiplier



- Built into cylindrical form factor
- pins at end are for various stage voltages and final stage readout pin

that's why it's called a photo-multiplier tube (PMT)

can be used to detect individual photons

Matter waves

light is a wave with f, λ wave properties

Planck & Einstein: $\boxed{E = hf}$ energy

$\Rightarrow$ looks like a particle that we call a "photon"

relativity: $E^2 = (pc)^2 + (m_0 c^2)^2$

mass of photon $= 0$ experimentally

so $E = pc = hf$

$$\boxed{p = \frac{hf}{c} = \frac{h}{\lambda}}$$

Compton scattering assumes light is a particle and experiments confirm Compton scattering

so a wave = a particle

$$\left.\begin{aligned} E &= hf = \frac{h}{2\pi} \cdot 2\pi f = \hbar\omega \\ p &= \frac{h}{\lambda} = \frac{2\pi}{\lambda} \cdot \frac{h}{2\pi} = \hbar k \end{aligned}\right\} \hbar = \frac{h}{2\pi}$$

$$\hbar c = \frac{hc}{2\pi} = \frac{1243 \text{ eV-nm}}{2\pi} = 197.8 \text{ eV-nm}$$

does a particle w/ mass also have wave properties?

yes! then if a particle has m, E, p

then $f = \frac{E}{h}$ & $\lambda = \frac{h}{p}$ for particles (electrons, protons, ...)

here E & p are the usual energy & momentum (and can be relativistic or not)

ex: baseball mass ~ 145 gm

velocity ~ 90 mph * $\frac{0.447 \text{ m/s}}{1 \text{ mph}}$ = 40.2 m/s

$v \ll c$ so use non-relativistic

$p = mv > 0.145 \text{ kg} * 40.2 \text{ m/s} = 5.83 \text{ kg m/s}$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J-s}}{5.83 \text{ kg} \frac{m}{s}} = 1.14 \times 10^{-34} \text{ m}$$

too small to measure!

so wave properties are irrelevant for "classical" every day experience

ex: electron has $E = 2 \text{ MeV} > m_0 c^2$

$$E = hf = \frac{hcf}{c} \Rightarrow f = \frac{E \cdot c}{hc}$$

$$= \frac{2 \times 10^6 \text{ eV}}{1243 \text{ eV-nm}} * 3 \times 10^8 \frac{m}{s} * \frac{1 \text{ nm}}{10^{-9} \text{ m}}$$

$$= 4.8 \times 10^{20} \text{ Hz!}$$

electron mass = $m_0 c^2 = 0.511 \text{ eV} < E$ so use relativity to get electron momentum use $E^2 = (pc)^2 + (M_o c^2)^2$

$$pc = \sqrt{E^2 - (m_o c^2)^2} = \sqrt{(2\,MeV)^2 - (0.511\,MeV)^2}$$

$$= 1.93\ MeV$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1243\ eV\text{-}nm}{1.93 \times 10^6\ eV} = 6.4 \times 10^{-4}\ nm$$

$$= 0.64 \times 10^{-3}\ nm$$

$$= 0.64\ pm$$

Note: for particles: $E = hf$

$$p = \frac{h}{\lambda}$$

so $\frac{E}{p} = \frac{hf}{h/\lambda} = \lambda f = v$ or $\frac{E}{pc} = \frac{v}{c} = \beta$

but if you write $E = \gamma m_o c^2$

$$pc = \gamma \beta m_o c^2$$

then $\frac{pc}{E} = \beta$ or $\frac{E}{pc} = \frac{1}{\beta}$

so which is correct?

this brings up difference between the

v_g = group velocity

v_p = phase velocity

v_g carries energy, not v_p so $\frac{E}{pc} = v_g$ group velocity

and $v_p = \lambda f$ phase velocity

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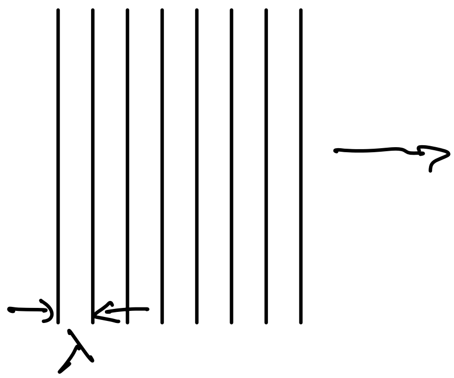
Waves & particles

waves have wavelength $k = \frac{2\pi}{\lambda}$

wavefront :



has definite wavelength

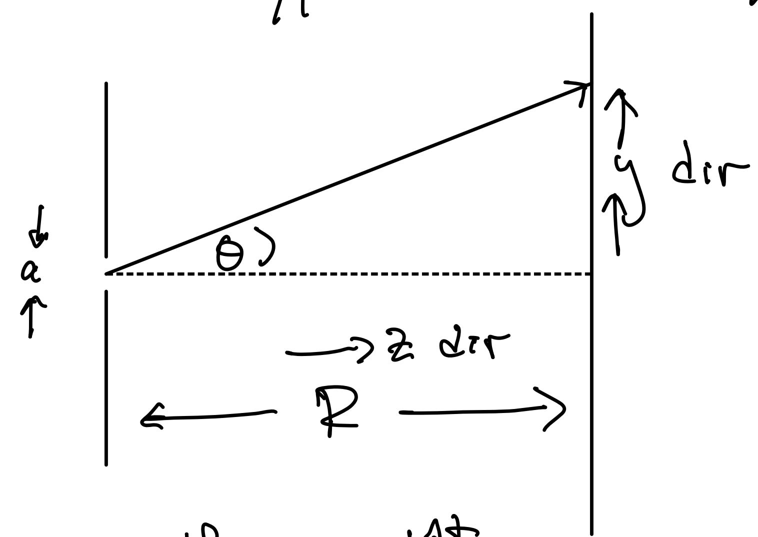
but is not localized $\rightarrow$ wave exists along infinite wavefront

particles have position coordinates

$\Rightarrow$ but a particle not moving does not have a definite wavelength

$\Rightarrow$ this implies that in wave-particle picture there is a relationship between the degree that you can know position: uncertainty in $x \Rightarrow \Delta x$
and momentum: " " $p \Rightarrow \Delta p$
(thus $p = h/\lambda$ relation)

Single slit diffraction & wave/particle



from wave theory: 1^{st} minimum is at

$$a \sin\theta_1 = \lambda$$

uncertainty in wave position in slit:

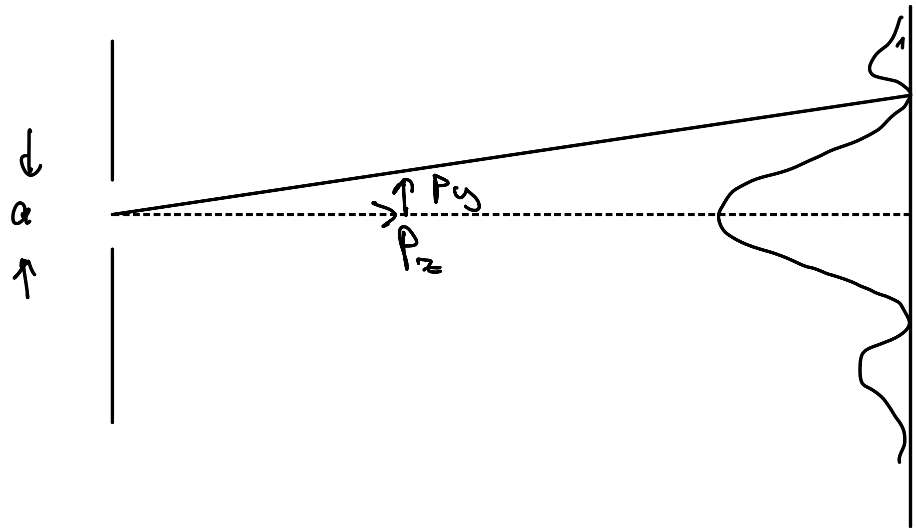
$$\Delta y = a$$

wave has a momentum along z direction:

$$p_z = \frac{h}{\lambda} = \hbar k$$

for photon to get to 1^{st} min angle, it would have to pick up some unknown momentum Δp_y

and can write $\sin\theta \sim \tan\theta = \dfrac{\Delta p_y}{p_z}$



so $\quad a\sin\theta \to \Delta y \cdot \dfrac{\Delta p_y}{p_z} = \lambda$

or $\quad \Delta y\, \Delta p_y \sim \lambda p_z = \lambda \dfrac{h}{\lambda} = h$

$$\Delta y\, \Delta p_y \sim h$$

⇒ when you do the calculation using more modern techiques of quantum mechanics you get:

$$\Delta x\, \Delta p \geq \hbar/2$$

Δx is "uncertainty" in position

Δp " " " momentum along x

this kind of relationship is common in Fourier analysis of signals

⇒ for wave/particles, it all has to do with the idea that a wave is <u>not</u> a localized thing <u>but a particle is</u>

current theory on what is a matter wave:

wave function for matter wave: $\psi(x,t)$

this wave can be a simple periodic oscillation:

$\psi(x,t) = A\sin(kx-\omega t)$ for a free particle

it describes a particle w/ wavelength $k = \frac{2\pi}{\lambda}$
and freq $\omega = 2\pi f$

- but what is oscillating?

$\Rightarrow$ ψ is the "amplitude" for finding the particle at position x, at time t

$\Rightarrow$ $|\psi|^2$ is the <u>probability</u> of finding the particle there & then

- where is the particle if it has a wavelength and frequency?

$\Rightarrow$ that's where uncertainty relation comes in:

if you constrain particle to be in region Δx, then you cannot know its momentum precisely

$\Rightarrow$ $\Delta p = \frac{\hbar/2}{\Delta x}$ so as $\Delta x \to 0$, $\Delta p \to \infty$ and
as $\Delta x \to \infty$, $\Delta p \to 0$

Heisenberg $\Delta x \, \Delta p_x \geq \hbar/2$

Very fundamental to QM:

⇒ you can not (in principle) know precisely both position and momentum

also there's an uncertainty relation for energy:

$$\Delta E \, \Delta t \geq \hbar/2$$

⇒ cannot localize in time and know energy precisely

The more localized in position, the less definite the momentum

& Vice versa

But the theory also says more:

- The position & momentum do not exist until measured!
- only the probability for having some value of position and momentum exist
- The wave equation tells you this probability

It is not that you <u>don't</u> know position & momentum

⇒ its that a definite position & a definite momentum do <u>not</u> exist

reality at subatomic (quantum) level:

the only thing that exists is probability

⇒ This might seem odd or wrong but <u>all</u> technology is based on it

Schrodinger's Cat

put cat in box w/ radioactive source

QM says:

1. cannot predict when source will decay
2. can predict probability as function of time that has decayed
3. can make many measurements, construct probability, compare w/ theory - agrees

4. before measuring, decay is not a valid concept -> particle is in a superposition of "decayed" and "not decayed"

=> state of decayed or not only is real once you make the measurement

=> this is called "collapse of wave function"

Schrodinger cat -> we still don't agree about it!